

Verify that the divergence Theorem is true for the vector field \mathbf{F} on the region E .

1) $\mathbf{F}(x, y, z) = 3x\mathbf{i} + xy\mathbf{j} + 2xz\mathbf{k}$, E is the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.

2) $\mathbf{F}(x, y, z) = x^2\mathbf{i} + xy\mathbf{j} + z\mathbf{k}$, E is the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy -plane.

Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, that is, calculate the flux of \mathbf{F} across S .

3) $\mathbf{F}(x, y, z) = e^x \sin y \mathbf{i} + e^x \cos y \mathbf{j} + yz^2 \mathbf{k}$, S is the surface of the box bounded by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, and $z=2$.

4) $\mathbf{F}(x, y, z) = 3xy^2 \mathbf{i} + xe^z \mathbf{j} + z^3 \mathbf{k}$, S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x=-1$ and $x=2$.

5) $\mathbf{F}(x, y, z) = x^2y\mathbf{i} + xy^2\mathbf{j} + 2xyz\mathbf{k}$, S is the surface of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + 2y + z = 2$.

6) $\mathbf{F}(x, y, z) = (x^3 + y \sin z)\mathbf{i} + (y^3 + z \sin x)\mathbf{j} + 3z\mathbf{k}$, S is the surface of the solid bounded by the hemispheres $z = \sqrt{4 - x^2 - y^2}$, $z = \sqrt{1 - x^2 - y^2}$ and the plane $z = 0$.